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LETTER TO THE EDITOR

Lorentz invariance of the quantum Hall effect and the finite frequency effects

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Abstract. It is shown that the macroscopic equations describing the quantum Hall effect are Lorentz invariant. The influence of finite frequency on the symmetries underlying the QHE is studied and the role of the polarisation currents in the plateau regime is discussed.

In the theory of continuous media (Landau and Lifshitz 1960) the induced charges and currents are linearly connected with electric and magnetic fields:

$$j_\mu = \Sigma_{\mu\nu\rho} F_{\nu\rho} \tag{1}$$

where j_μ is the current, $F_{\nu\rho}$ is the electromagnetic field strength tensor, and $\Sigma_{\mu\nu\rho}$ are the coefficients. The relativistic invariance is preserved if the coefficients $\Sigma_{\mu\nu\rho}$ form a constant tensor. In $D + 1$ space–time dimensions, the only constant tensors are the metric tensor $g_{\mu\nu}$ and the totally antisymmetric (D + 1)-component tensor $\varepsilon_{\mu\nu}\ldots$. In 3 + 1 dimensions, it is impossible to construct a constant three-component tensor out of $g_{\mu\nu}$ and $\varepsilon_{\mu\nu\rho\sigma}$; therefore the 3D macroscopic electrodynamics is not Lorentz invariant. However, in 2 + 1 dimensions such a tensor may exist: $\Sigma_{\mu\nu\rho} = \text{constant} \times \varepsilon_{\mu\nu\rho}$.

We will show here that this may take place in a two-dimensional electron gas under quantum Hall effect (QHE) conditions (for a review on QHE see Prange and Girvin 1986). Namely, we show that the currents and the fields in the QHE are connected by a (2 + 1)-dimensional vector equation, equation (1), where

$$\Sigma_{\mu\nu\rho} = \nu\alpha\varepsilon_{\mu\nu\rho}. \tag{1a}$$

Here ν is the filling factor of the Landau levels and α is the fine structure constant.

In the QHE, the currents and fields are connected as follows:

$$j_1 = \nu\alpha E_2 \tag{2}$$

$$j_2 = -\nu\alpha E_1 \tag{3}$$

$$j_0 = \nu\alpha B_3. \tag{4}$$

Equations (2) and (3) are Ohm's law, and equation (4) is the differential form of

$$Q = \int j_0(x_1, x_2) = \nu\alpha \int B_3(x_1, x_2) = \nu\alpha\Phi \tag{4a}$$

where Φ is the magnetic flux. Physically it reflects the fact that in a non-dissipative medium the electrons are 'glued' to magnetic force lines (the Faraday law).

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Let us discuss now the time evolution of currents and fields in a QHE situation, figure 1. An axially symmetric fluctuation of the magnetic flux $\delta\Phi(t, \boldsymbol{r})$ will drive a charge density fluctuation $\delta n(t, \boldsymbol{r})$ which, in turn, will create a radial electric field $E(t, \boldsymbol{r})$. Finally a magnetic moment $\delta \boldsymbol{M}(t, \boldsymbol{r})$ will be added to the initial magnetic field, due to the azimuthal Hall currents: $j_\varphi(t, \boldsymbol{r}) \propto E_r H_z$. This may result in an oscillatory process, i.e. in a periodic transformation of the magnetic energy into the kinetic energy of the Hall currents and vice versa, if the energy stored in such a fluctuation will dissipate (due to the Joule heat produced by the radial currents $W \propto \int j_r E_r$) slowly with respect to the oscillation period.

To show this we start with the continuity equation

$$- e\, \mathrm{d}\delta n/\mathrm{d}t + \operatorname{div} \delta \boldsymbol{j} = 0 \tag{5}$$

where the current is defined as

$$j_i = \sigma_{ij} E_j. \tag{6}$$

In the QHE, $\sigma_{xx} = 0$ and equation (5) reads

$$- e\, \mathrm{d}\delta n/\mathrm{d}t = \operatorname{div}(\sigma_{xy}(\delta \boldsymbol{E} \times \boldsymbol{h})) = -\sigma_{xy} \boldsymbol{h} \operatorname{curl} \delta \boldsymbol{E} + (\delta \boldsymbol{E} \times \boldsymbol{h}) \operatorname{grad} \sigma_{xy}. \tag{5a}$$

Using the Faraday law

$$c \operatorname{curl} \boldsymbol{E} = -\mathrm{d}\boldsymbol{B}/\mathrm{d}t \tag{7}$$

we arrive at

$$- e\, \mathrm{d}\delta n/\mathrm{d}t = -(\sigma_{xy}/c)(\mathrm{d}\delta \boldsymbol{B}/\mathrm{d}t) + (\delta \boldsymbol{E} \times \boldsymbol{h}) \operatorname{grad} \sigma_{xy}. \tag{8}$$

Neglecting the non-linear terms we arrive at the desired proportionality between the magnetic flux and the charge density

$$\delta B = -(ec/\sigma_{xy})\delta n \tag{4b}$$

It follows, from equation (4*b*), that the vanishing of the diagonal components of the conductivity tensor, $\sigma_{xx} = 0$, ensures that the Hall conductivity σ_{xy}, the only material parameter entering the linearised equations, is a constant of motion. Moreover, the QHE fixes this constant at a universal value ie^2/h (where i is an integer, or a simple fraction p/q), thus giving a universal relationship between the charge density and the magnetic flux fluctuations. Markiewicz (1986) outlined the idea that the flux conservation in QHE is analogous to the magnetic flux quantisation in superconductors.

Equations (2)–(4) could be cast into a Lorentz invariant tensorial form:

$$j_\alpha = \nu(e^2/h) f_\alpha \tag{9}$$

where f_α is a vector dual to the field strength tensor $f_{\alpha\beta}$: $f_\alpha \equiv \varepsilon_{\alpha\beta\gamma} f^{\beta\gamma}$. Equation (9) is the only local covariant linear equation in 2 + 1 dimensions because f_α is the only vector that can be constructed from $f_{\alpha\beta}$ and constant tensors $g_{\alpha\beta}$ and $\varepsilon_{\alpha\beta\gamma}$.

The Lorentz symmetry depends crucially on the absence of diagonal components in the conductivity tensor in the QHE. The exactness of this symmetry, therefore, is limited by the 'degree of vanishing' of σ_{xx}. In what follows we identify a phenomenon that destroys the Lorentz invariance: polarisation currents appearing at finite frequencies.

The problem of the finite frequency response in the quantum Hall effect regime is attracting growing attention, since Pepper and co-workers reported (Pepper and Wakabayashi 1983; for a review see Pepper 1985) that the QHE plateaus may be destroyed by application of a relatively low frequency. Goldberg and co-workers have attributed

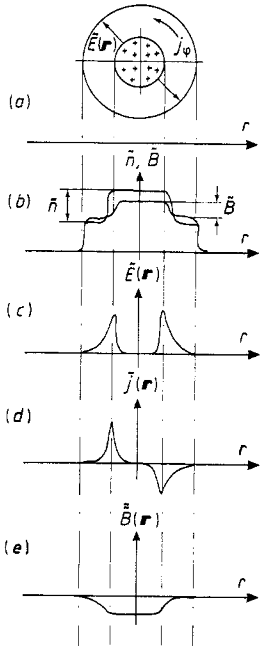

Figure 1. The time evolution of currents and fields in a QHE situation. (*a*) An axially symmetric fluctuation of the magnetic flux $\delta\Phi(t, \boldsymbol{r})$; (*b*) a charge density fluctuation $\delta n(t, \boldsymbol{r})$; (*c*) a radial electric field $E(t, \boldsymbol{r})$; (*d*) the azimuthal Hall currents: $j_\varphi(t, \boldsymbol{r}) \propto E_r H_z$; (*e*) a magnetic moment $\delta M(t, \boldsymbol{r})$ created by the azimuthal currents.

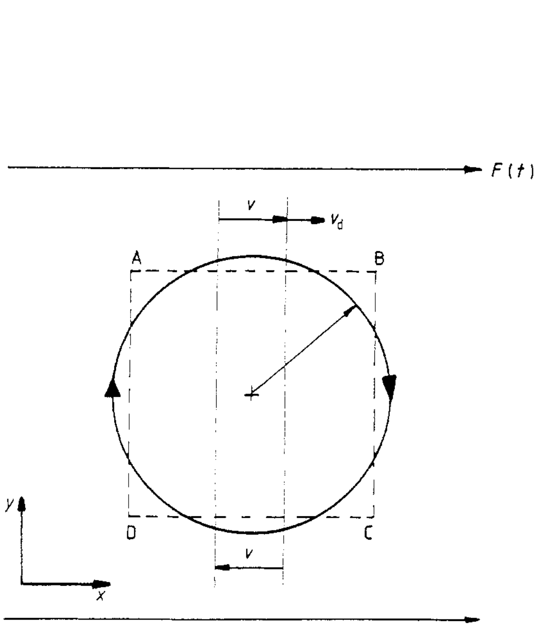

Figure 2. An electron experiences *time dependent* acceleration $F(t)/m$ on the arcs (approximated by straight lines) AB and CD, and acquires a drift velocity v_d along the electric field as defined in equation (10).

the frequency dependent effects to the coupling between the 2DEG and the gate material. Recently, Lee *et al* (1987) have shown that the effect is *genuine*, rather than due to the capacitative losses. They observed an almost linear frequency dependence of the diagonal conductivity minima and interpreted their results within the percolation picture (Iordansky 1982, Luryi and Kazarinov 1983, Joynt 1985, Apenko and Lozovik 1985, Rosenstein and Vagner 1989).

Vagner and Bergman (1987) have shown that in contrast to Re σ_{xx} vanishing in the plateau region, the imaginary part of the diagonal conductivity remains finite, due to the polarisation currents, and is linearly dependent on frequency: Im $\sigma_{xx} \propto (\omega/\omega_c)\sigma_{xy}$. The origin of the polarisation currents, contributing to Im σ_{xx}, is shown in figure 2. An electron experiences *time dependent* acceleration on the arcs (approximated by straight lines) AB and CD, and acquires a drift velocity along the electric field

$$V_{\text{pol}} \propto \frac{eE_0}{m} \int_{\text{A}}^{\text{B}} e^{i\omega t}\, dt \propto i \frac{\omega}{\omega_c^2} \frac{eE_0}{m} \tag{10}$$

and a corresponding current: $j_p = nev_p = nmc^2E/H^2$ in the direction of the time-varying electric field $E(t)$. This contributes to the diagonal conductivity

$$\sigma_{xx} = i\omega nmc^2/H^2 = i(\omega/\omega_c)\sigma_{xy}. \tag{11}$$

It was outlined by Vagner and Bergman (1987) that the wave propagation in a superlattice with the 2D electron gas in the quantum Hall regime may be used to study the influence of polarisation currents on the exactness of the plateaus, under conditions of helicon resonance. Helicon waves are caused by the Hall currents (Lifshitz and Pitaevskii 1981) and can be thought of essentially as a RF Hall effect, so plateaus in the helicon resonance, concomitant with the plateaus in σ_{xy}, should appear. Wendler and Kaganov (1987) and Narahari Achar (1988) studied the absorption of the helicon wave in a superlattice under QHE conditions. In a superlattice with a finite k_z-dispersion (minibands), temporal dispersion will cause dissipation and inclusion of the polarisation currents will result in non-local effects (Vagner 1977, 1982).

To summarise, we looked upon the known properties of 2D electron gas under strong magnetic fields from a novel point of view: the underlying symmetries of the macroscopic Maxwell equations. We found that in the plateau regions this system exhibits Lorentz and scaling symmetries, and estimated their exactness. We identified a possible symmetry breaking mechanism—polarisation currents in the AC QHE—and outlined how they can be measured in the helicon resonance in a superlattice with a 2D electron gas.

It is amusing that the only mathematically possible current–field interrelation exhibiting relativistic invariance, equation (1), for $D \leq 3$, has a solid state realisation: the quantum Hall effect. This remarkable system is, therefore, an example of a relativistic medium.

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